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*Short Note*

# Comparative quadrupole moments of triaxial superdeformed states in $^{163,164,165}$Lu

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**Abstract.** Average transition quadrupole moments in the yrast triaxial superdeformed bands of $^{163}$Lu, $^{164}$Lu and $^{165}$Lu were determined in a Doppler-shift attenuation-method experiment. Fractional Doppler shifts were determined in $\gamma$-ray coincidence spectra measured with the Gammasphere array. The transition quadrupole moments derived from these data show a decrease from $^{163}$Lu to $^{165}$Lu which is not predicted by total-energy surface calculations.

**PACS.** 21.10.-k Properties of nuclei; nuclear energy levels – 21.10.Tg Lifetimes – 25.70.-z Low and intermediate energy heavy-ion reactions – 27.70.+q $150 \leq A \leq 189$

In the mass region around $A = 165$ a large number of rotational bands have been discovered [1–8] over the past years which are associated with calculated potential-energy minima of large deformation ($\epsilon \approx 0.4$) and large triaxiality ($\gamma \approx \pm 20°$). Such triaxial superdeformed (TSD) states had already been predicted theoretically more than 10 years ago [9]. For some of these bands the large deformation has been verified by lifetime measurements [2,7,10] and, in the case of $^{163}$Lu, the triaxiality has recently been proven by the discovery of the wobbling mode [8,11–13]. New systematic total-energy surface calculations with the Ultimate Cranker (UC) code [14,15] predict that TSD minima exist at high spins in the whole region of nuclei around $Z \approx 72$ and $N \approx$ 90–98. These minima appear for all four combinations of parity and signature. The quadrupole moments that are obtained directly from the wave functions [15] vary between 8.5 b and 11.5 b for positive gamma values and between 10 b and 15 b for negative gamma. Correspondingly, the energy minima with positive gamma values lie in the deformation range $0.36 < \epsilon < 0.44$ and $16° < \gamma < 23°$, and those with negative gamma lie in the range $0.35 < \epsilon < 0.43$ and $-23° < \gamma < -15°$. On the other hand, the experimental quadrupole moments seem to exhibit a larger variation for the different isotopes [2,7,10] than predicted theoretically.

The purpose of this work was to perform a comparative measurement of the transition quadrupole moments of the yrast TSD bands in three Lu isotopes, $^{163}$Lu, $^{164}$Lu and $^{165}$Lu. All three isotopes have been produced in the same reaction and the lifetimes could be determined under identical conditions. Thus, even if the absolute values might be subject to rather large uncertainties, the relative lifetimes, and the relative quadrupole moments derived from them, are more reliable. Lifetimes have been measured previously for the TSD bands 1 in $^{163}$Lu and $^{164}$Lu [2,10], but no values existed for $^{165}$Lu.

High-spin states in $^{163}$Lu, $^{164}$Lu and $^{165}$Lu were populated in the reaction $^{124}$Sn($^{45}$Sc$, x$n), the $^{45}$Sc beam of 217 MeV being provided by the 88-Inch cyclotron at LBNL. The target consisted of a foil of enriched $^{124}$Sn with a thickness of 1 mg/cm$^2$ on a 13 mg/cm$^2$ thick gold backing to stop the recoiling nuclei. The Sn and Au foils were separated by a 95 $\mu$g/cm$^2$ thick Ta layer to prevent amalgamation at the boundary between Sn and Au when the target is heated by the beam. Gamma-ray coincidences were measured with the Gammasphere spectrometer array which consisted of 100 Compton-suppressed Ge detectors

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**Table 1.** Experimental transition and side-feeding quadrupole moments for the yrast TSD bands in $^{163,164,165}$Lu. The values in the left two columns are determined by measuring the transition energies at different angles (method (a)). The two columns in the middle show the values extracted by overlaying the spectra from detectors at the various angle groups for different $F(\tau)$ values (method (b), see text).

| Isotope | Method (a) | | Method (b) | | Previous work [10] | |
|---|---|---|---|---|---|---|
| | $Q_\mathrm{t}(b)$ | $Q_\mathrm{sf}(b)$ | $Q_\mathrm{t}(b)$ | $Q_\mathrm{sf}(b)$ | $Q_\mathrm{t}(b)$ | $Q_\mathrm{sf}(b)$ |
| $^{163}$Lu | $7.4^{+0.7}_{-0.4}$ | $6.7^{+0.7}_{-0.7}$ | $7.7^{+2.3}_{-1.3}$ | $7.0^{+0.7}_{-0.7}$ | $8.2^{+1.0}_{-0.6}$ | $7.5^{+0.8}_{-0.8}$ |
| $^{164}$Lu | | | $7.4^{+2.5}_{-1.3}$ | $6.7^{+0.7}_{-0.7}$ | $7.1^{+0.5}_{-0.6}$ | $6.4^{+0.6}_{-0.6}$ |
| $^{165}$Lu | $6.0^{+1.2}_{-0.2}$ | $5.4^{+0.5}_{-0.5}$ | $6.4^{+1.9}_{-0.7}$ | $5.8^{+0.6}_{-0.6}$ | | |

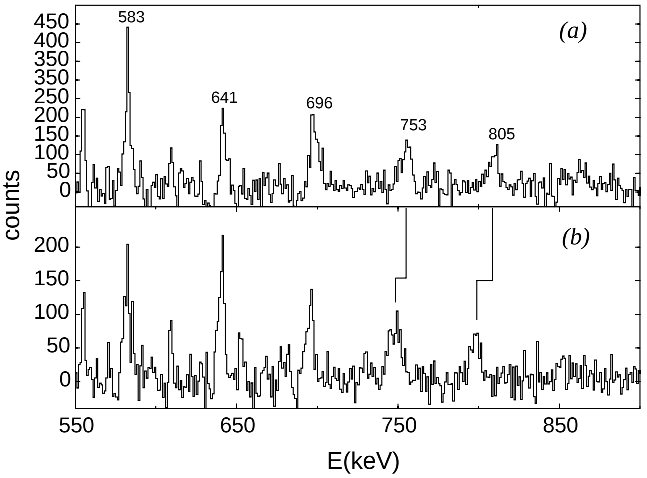

**Fig. 1.** Double-gated spectra from detectors in different angle groups ((a) forward, (b) backward) for TSD band 1 in $^{165}$Lu.

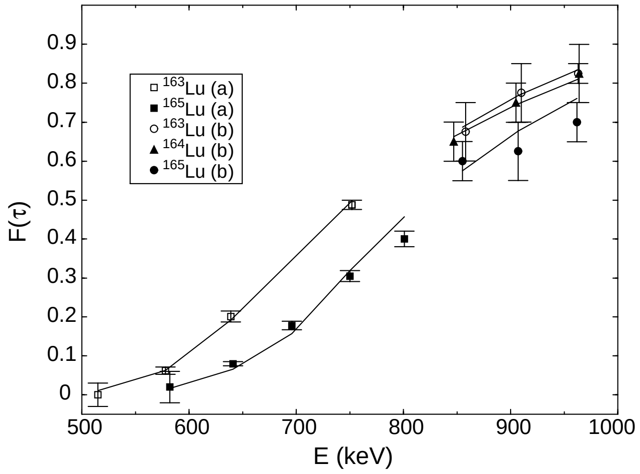

**Fig. 2.** Fractional Doppler shifts $F(\tau)$ for TSD bands 1 in $^{163,164,165}$Lu. The curves are least-square fits to the data points. Data obtained by the two methods (a) and (b) are shown (see text).

at the time of the experiment. Coincidence events were written to magnetic tape when four or more of the Ge detectors showed a signal. This resulted in $2 \times 10^9$ events which were sorted into a BLUE data base [16].

For the analysis, the Ge detectors were grouped into 15 rings at the following angles with respect to the beam direction: 17.27°, 31.72°, 37.38°, 50.07°, 58.28°, 69.82°, (79.19° and 80.71°), 90°, (99.29° and 100.81°), 110.18°, 121.72°, 129.93°, 142.62°, 148.28°, 162.73°. Coincidence spectra were produced from the data base by setting gates on uncontaminated $\gamma$-ray transitions measured in all detectors and incrementing spectra for the detectors in the different angle groups. Only spectra obtained with coincidence gates below the transitions for which the lifetimes were analysed had sufficient statistics. As example, fig. 1 shows spectra from the forward and backward detectors for TSD band 1 in $^{165}$Lu. Spectra from the near-90° detectors were used to locate possible contaminations at the positions of the shifted peaks in the spectra from the forward and backward detectors.

Fractional Doppler shifts, $F(\tau)$, were analysed for the yrast TSD bands in $^{163}$Lu and $^{165}$Lu. From the centroids of the $\gamma$-ray lines determined in the spectra measured at the different angles with respect to the beam direction the fractional shifts $F(\tau) = (\langle E_\gamma(\theta)\rangle - E^0_\gamma)/(E^0_\gamma \cdot \beta_\mathrm{max} \cdot \cos(\theta))$ are obtained. Here, $E^0_\gamma$ is the energy of the unshifted line and $\beta_\mathrm{max} = v_\mathrm{max}/c = 0.0268$ with the initial velocity $v_\mathrm{max}$ calculated from the reaction parameters for the center of the target. The centroid of the Doppler-shifted peak $\langle E_\gamma(\theta)\rangle$ depends on the angle $\theta$ relative to the beam direction. $F(\tau)$ is determined by a linear fit to the experimental shifted peaks $\langle E_\gamma(\theta)\rangle$ plotted as a function of $\cos(\theta)$. These $F(\tau)$ values are shown as a function of the $\gamma$-ray energy (between 500 and 810 keV) in fig. 2.

$F(\tau)$ curves were calculated using a program developed by Moore *et al.* [17]. It assumes a rotational cascade with a constant transition quadrupole moment $Q_\mathrm{t}$. The side feeding is modeled by a rotational cascade with a constant quadrupole moment $Q_\mathrm{sf}$ and with a constant moment of inertia of $J^{(2)}$. For $J^{(2)}$, values are used which correspond to the moments of inertia of the yrast TSD bands in $^{163}$Lu and $^{165}$Lu in the frequency range where the lifetimes are analysed. The side-feeding intensities were determined from the measured $\gamma$-ray intensities within the bands. In our earlier work on lifetimes in Lu isotopes [10] we found that $Q_\mathrm{sf}$ is similar to $Q_\mathrm{t}$ by comparing spectra with gates above and below the states of interest. This is in agreement with a systematic comparison of side-feeding and in-band lifetimes [18] which showed that $\tau_\mathrm{sf} \approx 1.2\tau_\mathrm{band}$. Therefore, we adopted $Q_\mathrm{sf} = 0.91 Q_\mathrm{t}$ and allowed for a

10% variation in order to determine this contribution to the systematic uncertainties of the transition quadrupole moments. The electronic stopping powers were calculated using the tables of Ziegler *et al.* [19] and the nuclear stopping was treated according to the theory of Lindhard *et al.* [20].

The calculated $F(\tau)$ curves were fitted to the data with the quadrupole moments $Q_{\mathrm{t}}$ as free parameters. The best fits are shown in fig. 2 as solid lines. The results for the quadrupole moments are given in the first columns of table 1 (method (a)). The quoted errors include the statistical uncertainties and the covariance between the fit parameters. They do not include the systematic uncertainties associated with the stopping powers.

The TSD band 1 in $^{164}$Lu [5] is only weakly populated and it was not possible to directly measure fractional Doppler shifts from the spectra measured at the individual angle groups. Therefore, an indirect method to determine these shifts was applied. Gamma-ray spectra measured at the various angles with respect to the beam direction were modified assuming different fractional Doppler shifts $F(\tau_i)$, corresponding to different state and side-feeding lifetimes. For each of these fractional shifts, the spectra from the 15 angle groups were added and the widths of the peaks in the sum spectra were inspected. Fixed intervals around the centroids of the peaks were integrated and the $F(\tau)$ value that resulted in the largest area was adopted (fig. 2, between 840 and 980 keV). The quadrupole moment was then calculated from these $F(\tau)$ values in the same way as explained for method (a) for the directly measured $F(\tau)$ values. The method was tested on the yrast TSD bands in $^{163}$Lu and $^{165}$Lu and the results agreed with the $F(\tau)$ values determined directly from the spectra. These quadrupole moments are summarized in the central columns of table 1 (method (b)). The larger errors reflect the larger uncertainties connected with the indirect method of determination of the fractional Doppler shifts.

The transition quadrupole moments for the yrast TSD bands in $^{163}$Lu and $^{164}$Lu are, within experimental uncertainties, in agreement with our previous results [10]. No previous lifetime measurement exists for the TSD band in $^{165}$Lu.

The Doppler shifts of $\gamma$-ray transitions in the three bands have been determined under identical experimental conditions and the energy and spin ranges are very similar. Therefore, the quadrupole moments deduced from these shifts can be compared with great reliability. In particular, the relative values are not subject to the uncertainties connected with the treatment of the stopping powers.

We observe that the quadrupole moments of the yrast TSD bands show a decrease from $^{163}$Lu to $^{165}$Lu of 20%. Such a decrease is not reproduced by the calculations [15]. These calculations give average quadrupole moments of 9.2 b and 11.5 b for positive and negative $\gamma$ deformation respectively, for all the three Lu isotopes. Our experimental quadrupole moments lie closer to the calculations for positive $\gamma$, which also give the deeper total-energy minimum for the configurations of the three bands [15]. However, the absolute experimental values are almost 30% smaller than the theoretical quadrupole moments.

The fact that neither the absolute deformation nor the trend towards smaller deformation with increasing neutron number is reproduced by the calculations is probably due to the insufficient knowledge of the positions of the deformation-driving intruder orbitals $\pi i_{13/2}$, $\pi h_{9/2}$ and $\nu i_{13/2}$. The standard potential parameters obviously do not place these orbitals at the correct positions. Thus, experimental results on the deformation, like the quadrupole moments determined in this work, may help to optimize the mean-field parameters.

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